1. The function f is such that $f''(x) = 2x - 2$.

When the graph of f is drawn, it has a minimum point at (3, –7).

(a) Show that $f'(x) = x^2 - 2x - 3$ and hence find $f(x)$.

(6)

(b) Find $f(0)$, $f(-1)$ and $f'(-1)$.

(3)

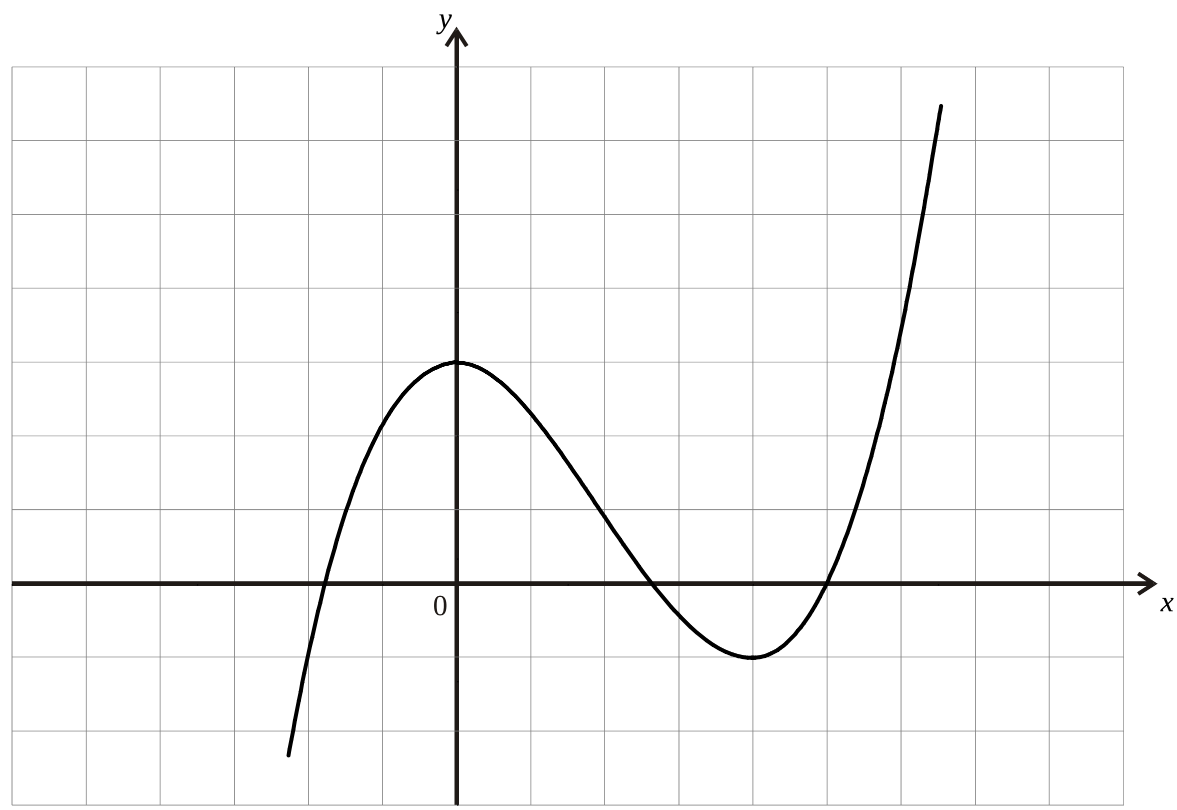
(c) Hence sketch the graph of f, labelling it with the information obtained in part (b).

(4)

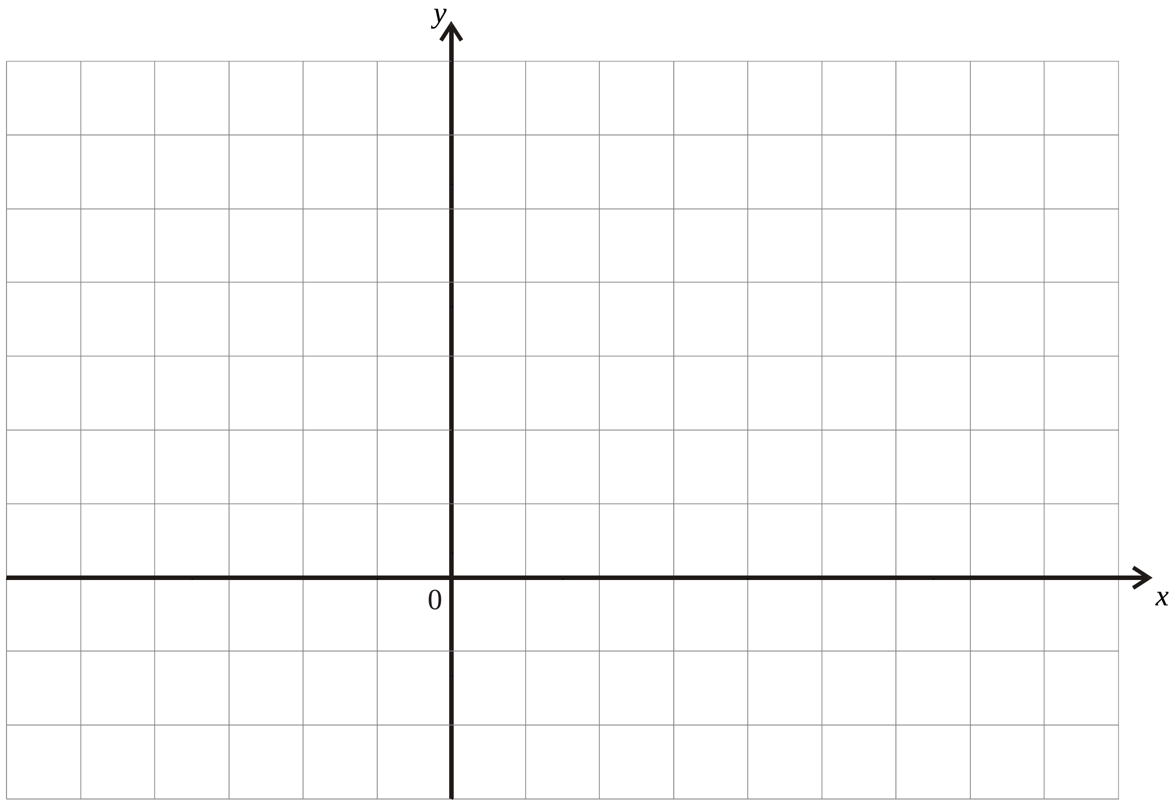
(**Note:** It is **not** necessary to find the coordinates of the points where the graph cuts the x-axis.)

(Total 13 marks)

2. The diagram shows the graph of $y = f(x)$.



On the grid below sketch the graph of $y = f'(x)$.



(Total 6 marks)

3. A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height h metres of the rock-climber after t seconds of the fall is given by:

$$h = 50 - 5t^2, \qquad 0 \le t \le 2$$

$$h = 90 - 40t + 5t^2, \qquad 2 \le t \le 5$$

(a) Find the height of the rock-climber when $t = 2$.

(1)

(b) Sketch a graph of h against t for $0 \le t \le 5$.

(4)

(c) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$ for:

(i) $0 \leq t \leq 2$

(ii) $2 \leq t \leq 5$

(2)

(d) Find the velocity of the rock-climber when $t = 2$.

(2)

(e) Find the times when the velocity of the rock-climber is zero.

(3)

(f) Find the minimum height of the rock-climber for $0 \leq t \leq 5$.

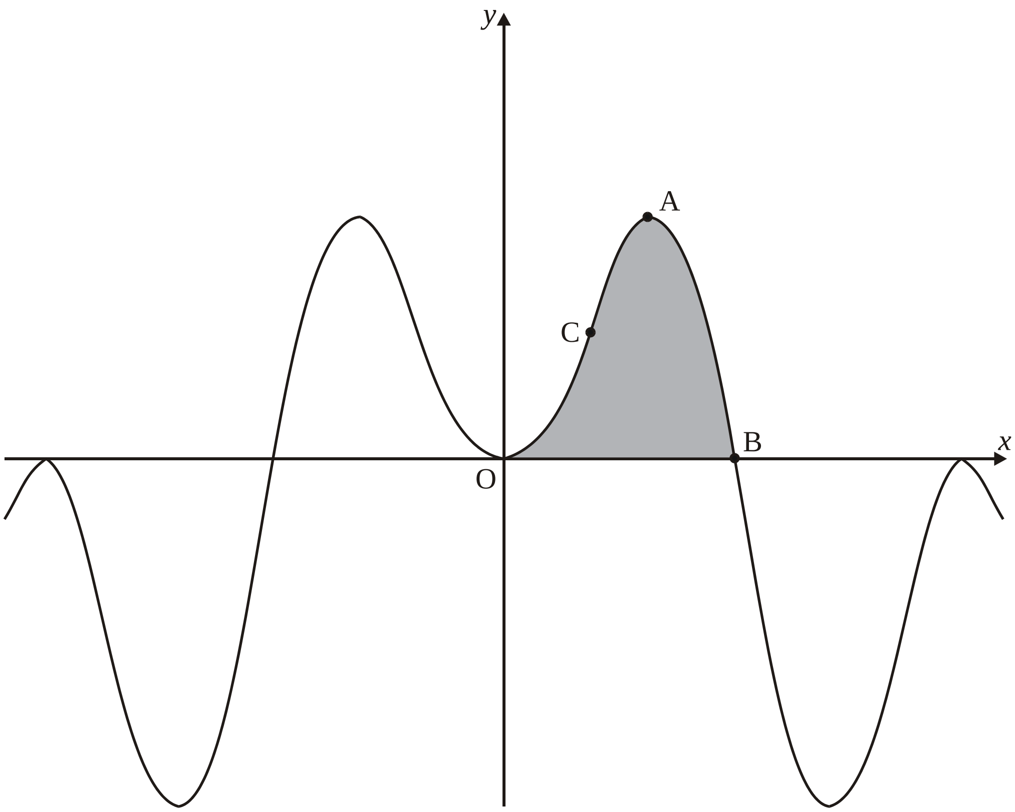
(3)
(Total 15 marks)

4. **In this part of the question, radians are used throughout.**

The function f is given by

$$f(x) = (\sin x)^2 \cos x.$$

The following diagram shows part of the graph of $y = f(x)$.



The point A is a maximum point, the point B lies on the x-axis, and the point C is a point of inflexion.

(a) Give the period of f.

(1)

(b) From consideration of the graph of $y = f(x)$, find **to an accuracy of one significant figure** the range of f.

(1)

(c) (i) Find $f'(x)$.

(ii) Hence show that at the point A, $\cos x = \sqrt{\frac{1}{3}}$.

(iii) Find the exact maximum value.

(9)

(d) Find the exact value of the x-coordinate at the point B.

(1)

(e) (i) Find $\int f(x)\,dx$.

(ii) Find the area of the shaded region in the diagram.

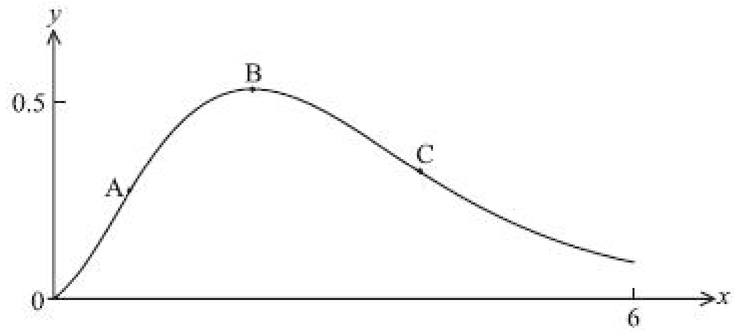
(4)

(f) Given that $f''(x) = 9(\cos x)^3 - 7\cos x$, find the x-coordinate at the point C.

(4)

(Total 20 marks)

5. The diagram below shows the graph of $f(x) = x^2 e^{-x}$ for $0 \le x \le 6$. There are points of inflexion at A and C and there is a maximum at B.



(a) Using the product rule for differentiation, find $f'(x)$.

(b) Find the **exact** value of the ***y*-coordinate** of B.

(c) The second derivative of f is $f''(x) = (x^2 - 4x + 2)\,e^{-x}$. Use this result to find the **exact** value of the x-coordinate of C.

....

....

....

....

....

....

....

....

....

....

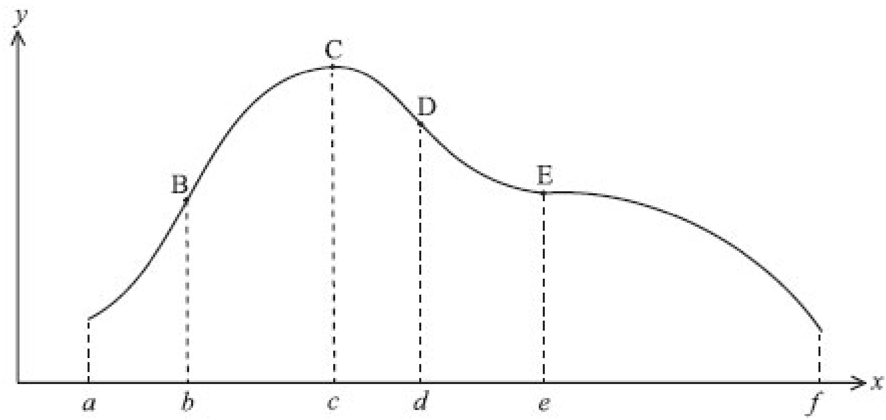
....

....

....

(Total 6 marks)

6. The graph of a function g is given in the diagram below.



The gradient of the curve has its maximum value at point B and its minimum value at point D. The tangent is horizontal at points C and E.

(a) Complete the table below, by stating whether the first derivative g' is positive or negative, and whether the second derivative g'' is positive or negative.

Interval	g'	g''
$a < x < b$		
$e < x < f$		

(b) Complete the table below by noting the points on the graph described by the following conditions.

Conditions	Point
$g'(x) = 0,\ g''(x) < 0$	
$g'(x) < 0,\ g''(x) = 0$	

(Total 6 marks)

7. Consider the function $f(x) = k \sin x + 3x$, where k is a constant.

(a) Find $f'(x)$.

(b) When $x = \frac{\pi}{3}$, the gradient of the curve of $f(x)$ is 8. Find the value of k.

Working:

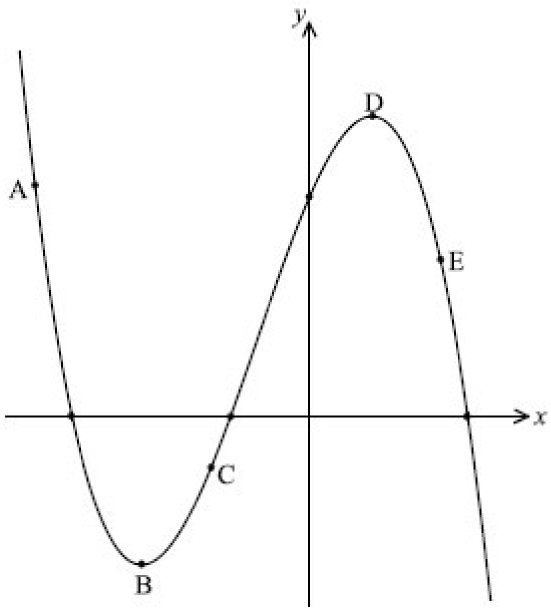
Answers:

(a) ....
...

(b) ....
...

(Total 4 marks)

8. The following diagram shows part of the curve of a function f. The points A, B, C, D and E lie on the curve, where B is a minimum point and D is a maximum point.



(a) Complete the following table, noting whether $f'(x)$ is positive, negative or zero at the given points.

	A	B	E
$f'(x)$			

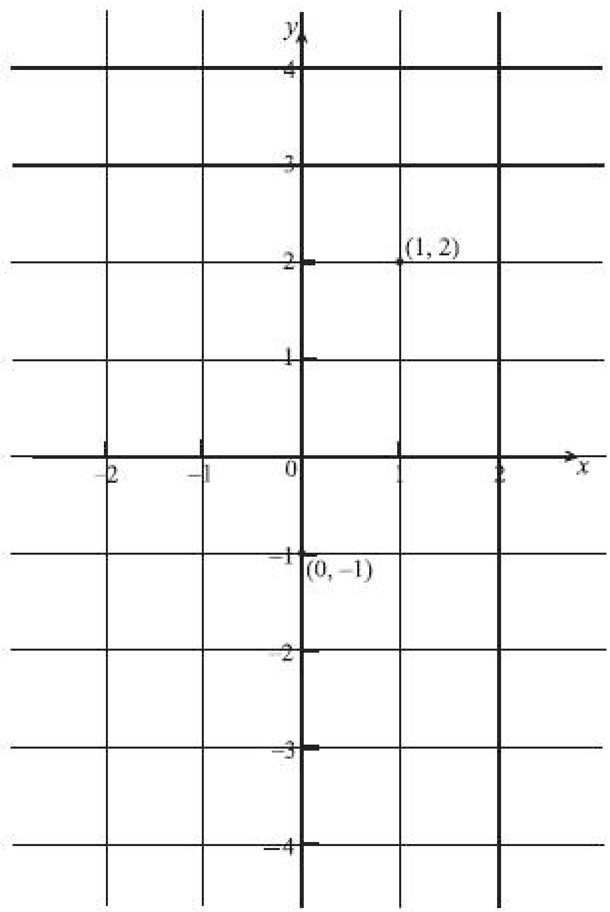
(b) Complete the following table, noting whether $f''(x)$ is positive, negative or zero at the given points.

	A	C	E
$f''(x)$			

(Total 6 marks)

9. On the axes below, sketch a curve $y = f(x)$ which satisfies the following conditions.

x	$f(x)$	$f'(x)$	$f''(x)$
$-2 \le x < 0$		negative	positive
0	–1	0	positive
$0 < x < 1$		positive	positive
1	2	positive	0
$1 < x \le 2$		positive	negative



(Total 6 marks)

10. *Radian measure is used, where appropriate, throughout the question.*

Consider the function $y = \dfrac{3x-2}{2x-5}$.

The graph of this function has a vertical and a horizontal asymptote.

(a) Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

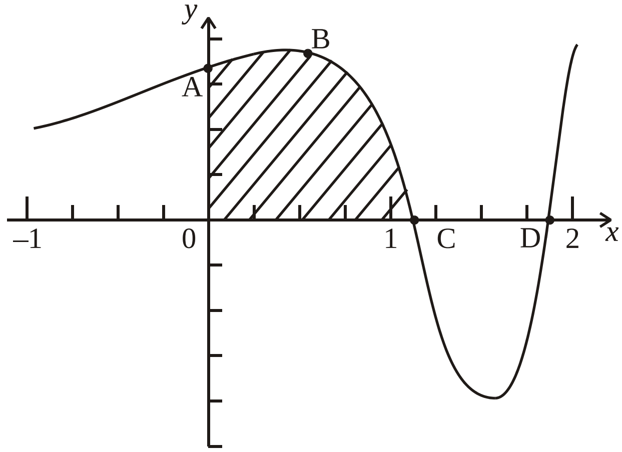
(b) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$, simplifying the answer as much as possible.

(3)

(c) How many points of inflexion does the graph of this function have?

(1)
(Total 6 marks)

11. The diagram below shows a sketch of the graph of the function $y = \sin(\mathrm{e}^x)$ where $-1 \le x \le 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.



(a) Find the coordinates of A.

(2)

(b) The coordinates of C may be written as (ln k, 0). Find the **exact** value of k.

(2)

(c) (i) Write down the y-coordinate of B.

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(iii) Hence, show that at B, $x = \ln \dfrac{\pi}{2}$.

(6)

(d) (i) Write down the integral which represents the shaded area.

(ii) Evaluate this integral.

(5)

(e) (i) Copy the above diagram into your answer booklet. (There is no need to copy the shading.) On your diagram, sketch the graph of $y = x^3$.

(ii) The two graphs intersect at the point P. Find the x-coordinate of P.

(3)
(Total 18 marks)

12. The function f is defined as $f(x) = (2x + 1)\,\mathrm{e}^{-x}$, $0 \le x \le 3$. The point P(0, 1) lies on the graph of $f(x)$, and there is a maximum point at Q.

(a) Sketch the graph of $y = f(x)$, labelling the points P and Q.

(3)

(b) (i) Show that $f'(x) = (1 - 2x)\,\mathrm{e}^{-x}$.

(ii) Find the **exact** coordinates of Q.

(7)

(c) The equation $f(x) = k$, where $k \in \mathbb{R}$, has two solutions. Write down the range of values of k.

(2)

(d) Given that $f''(x) = \mathrm{e}^{-x}(-3 + 2x)$, show that the curve of f has only one point of inflexion.

(2)

(e) Let R be the point on the curve of f with x-coordinate 3. Find the area of the region enclosed by the curve and the line (PR).

(7)

(Total 21 marks)

13. Let $f(x) = \mathrm{e}^{\frac{x}{3}} + 5\cos^2 x$. Find $f'(x)$.

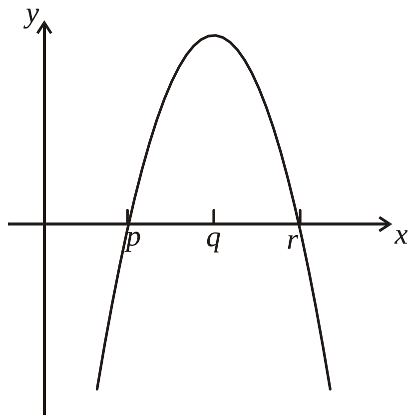
Working:

Answer:

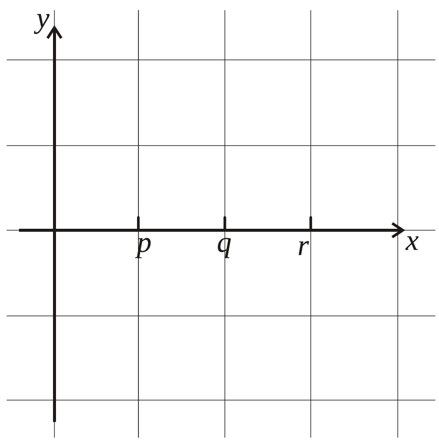
....

(Total 6 marks)

14. The diagram below shows part of the graph of the **gradient** function, $y = f'(x)$.



(a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept.



(2)

(b) Complete the table, for the graph of $y = f(x)$.

	x-coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

(2)

(c) Justify your answer to part (b) (ii).

....

....

....

....

....

(2)

(Total 6 marks)

15. Consider the function $f(x) = 1 + e^{-2x}$.

(a) (i) Find $f'(x)$.

(ii) Explain briefly how this shows that $f(x)$ is a decreasing function for all values of x (*ie* that $f(x)$ always decreases in value as x increases).

(2)

Let P be the point on the graph of f where $x = -\frac{1}{2}$.

(b) Find an expression in terms of e for

(i) the y-coordinate of P;

(ii) the gradient of the tangent to the curve at P.

(2)

(c) Find the equation of the tangent to the curve at P, giving your answer in the form $y = ax + b$.

(3)

(d) (i) Sketch the curve of f for $-1 \le x \le 2$.

(ii) Draw the tangent at $x = -\frac{1}{2}$.

(iii) Shade the area enclosed by the curve, the tangent and the y-axis.

(iv) Find this area.

(7)

(Total 14 marks)

16. **Figure 1** shows the graphs of the functions $f_1,\ f_2,\ f_3,\ f_4$.

Figure 2 includes the graphs of the derivatives of the functions shown in **Figure 1**, *eg* the derivative of f_1 is shown in diagram (d).

Figure 1

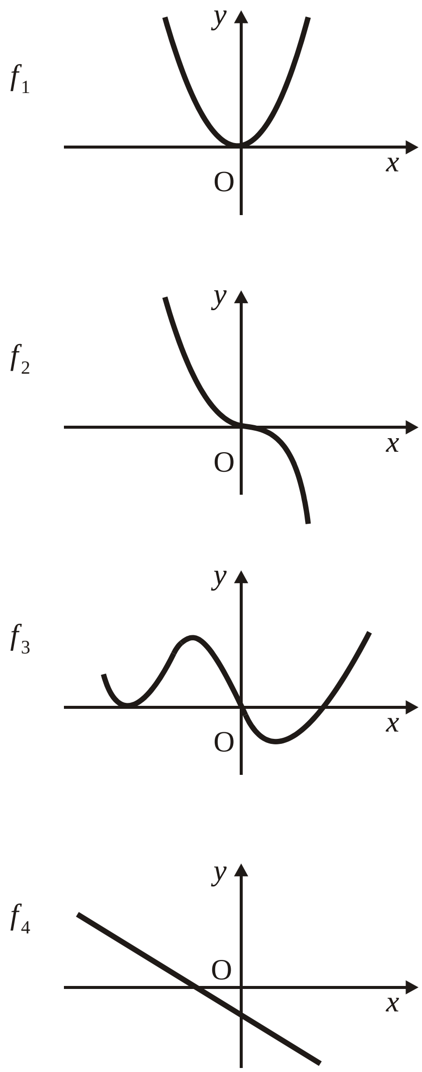
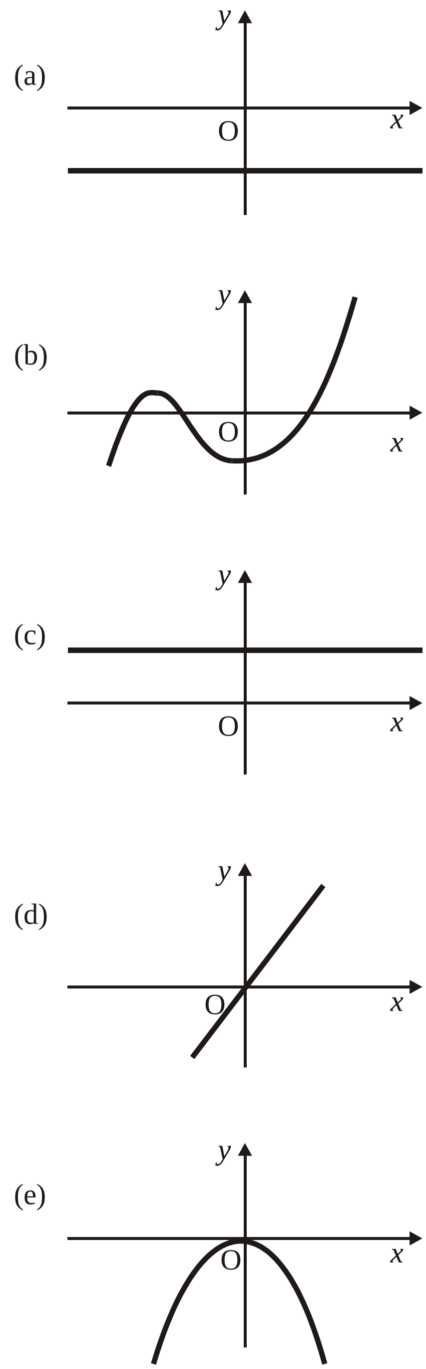


Figure 2



Complete the table below by matching each function with its derivative.

Function	Derivative diagram
f_1	(d)
f_2	
f_3	
f_4	

Working:

(Total 6 marks)